Experimental Study on the Propagation of Electromagnetic Waves and the Spatial Distribution of Electric Potential

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A scaling model experiment on the propagation of electromagnetic (EM) waves, which satisfies the attenuation with distance, has been proposed theoretically and carried out experimentally. A model consisting of parallel plate waveguides representing the earth and ionosphere, ocean and conductive layer in the crust is constructed using a solution of NaCl as the model crust and aluminum foils as the ocean both with resistivities, ρ, which are $10^{-6}$ times lower than those of the real ones. The frequency of the EM waves, $f$ was $10^6$ times higher than the real value so that the wavelength and the skin depth, $d = \sqrt{2\pi f/\rho}$ were scaled down to $10^{-6}$ in concordance with the scaling factor. A method using the electrolytic tank was used to determine the distribution of DC potential. The profiles of the EM waves and the DC potential were influenced by the presence of the ocean and faults of the Tokai area, taken as an example. A scaling-up experiment was suggested for the mesoscopic and nanosized optical and EM devices.

KEYWORDS: EM waves, earthquake, DC potential, electrolytic tank, waveguide, scaling, model, nanosized.

1. Introduction

Electromagnetic (EM) waves in a wide range of frequencies are said to have been observed near earthquake.\(^{1-5}\) Seismic electromagnetic signals (SEMS) of the DC earth potential named VAN method after the initials of three Greek scientists have been used for earthquake prediction.\(^{6,7}\) Propagation of EM waves can be described by the general equations of telecommunication and are scaled down in a model experiment using EM waves having small wavelengths.

A model of an electrically inhomogeneous earth considering an earth-crust and earth-ionosphere is composed of parallel plate waveguides as reported for a scaling model experiment carried out for the Kobe earthquake in 1995.\(^{8}\) Microwaves simulating the SEMS were emitted from the model hypocenter and the spatial distribution of intensity was mapped over the model crust of granite slab. The wave ripple of EM waves was formed by the standing waves at a higher frequency, while the exponential decay of evanescent waves was obtained at a frequency lower than the cut off frequency of the waveguide. However, the spread of EM waves was considerable since the skin depth of the crust was not scaled down to $10^{-6}$, the scaling factor of the map. The attenuation of EM waves in previous works was not scaled accurately since a granite slab was used as to represent the earth crust.

An electrostatic potential measurement method using an electrolytic tank has been used to simulate the potential in a vacuum tube. Arguments have been presented that geological faults are conductive due to the presence of electrolytic water at the fault planes. Hence, a simple method of experimental potential simulation has been applied to the model of the earth so that the earth potential depending on the local inhomogeneity of conductivity can be simulated.

In the present work, propagation of EM waves has been obtained using a novel model composed of materials having lower resistivities than those of real ones thereby satisfying the scaling factor of the skin depth and the Maxwell equation. The skin depth was scaled down to $10^{-6}$ in concordance with the model scale of $1:10^6$ using a salt solution for the model crust. The distribution of DC potential has been obtained using a classical electrolytic tank. A geographic model of the Tokai area in Japan was fabricated to obtain the intensity map of EM waves and DC potential as an example. A suggestion was made for a model experiment of mesoscopic and nanosized devices.

2. Theory of Scaling and Waveguide

2.1 The scaling relationship

The Maxwell equation for the propagation of EM waves in a conductive media with resistivity, $\rho$ is given as

$$\frac{\partial^2 E}{\partial z^2} - \frac{\mu}{\rho} \frac{\partial E}{\partial t} - \frac{\epsilon}{\rho} \frac{\partial^2 E}{\partial t^2} = 0,$$

where $E$ is the electric field, $\mu$ the magnetic permeability, and $\epsilon$ the dielectric constant.

If the time and the size are scaled down to $z_s = z/\gamma, t_m = t/\gamma$, the equation is reduced to

$$\frac{1}{\gamma^2} \left( \frac{\partial^2 E}{\partial z^2_s} - \frac{\mu \gamma}{\rho} \frac{\partial E}{\partial t_m} - \frac{\epsilon \gamma}{\rho} \frac{\partial^2 E}{\partial t^2_m} \right) = 0.$$

(2)

Hence, if one selects $\rho_m = \rho/\gamma$, the above equation is reduced to

$$\frac{1}{\gamma^2} \left( \frac{\partial^2 E}{\partial z^2_s} - \frac{\mu \gamma}{\rho_m} \frac{\partial E}{\partial t_m} - \frac{\epsilon \gamma}{\rho_m} \frac{\partial^2 E}{\partial t^2_m} \right) = 0.$$

(3)

This indicates that one can obtain the same value of $E$ in a model of the size and frequency which are scaled down to $1/\gamma$ and $\gamma$ ($\gamma = 10^6$ for example), respectively, from the original values if one selects some materials with a resistivity of $\rho_m = \rho/\gamma$. In fact, the general skin depth equation is given by

$$d = -1/\Im \left( \frac{\epsilon - i \mu}{\rho \omega} \right),$$

(4)

and the above scaling relationships give

$$d_m = -1/\Im \left( \frac{\epsilon - i \mu}{\rho_m \omega_m} \gamma \alpha_m \right) = \frac{d}{\gamma}.$$

(5)
The decay of the EM waves with the distance is also scaled down for a material $\rho$ which is $1/\gamma$ times the size of the earth since the scaling should satisfy the general equations of telecommunication. This indicates the accuracy of the intensity profile for a model experiment using a map scaled down to $1/\gamma$.

2.2 Propagation of EM waves in earth waveguides

The separation of the conductive parallel plates, $d$, and the speed of light, $c$, give a cut-off frequency of the $TE_1$ mode, $f_c = c/2d$, below which the EM waves become evanescent. Note that the near field case for a wavelength longer than that of the spacing between the plates can be expressed as evanescent waves in waveguide terminology.

The wavelength in a parallel plate waveguide, $\lambda_g$, is given by $1/\lambda_g^2 = 1/\lambda^2 - 1/2d^2$, where $\lambda = c/f$ is wavelength in free space or in the dielectric earth ($\lambda = c/f(e^{1/2})$) and $d$, is the separation between the parallel plates. For $\lambda > 2d$, i.e., $f < f_c = c/2d$, $\lambda_g$ is an imaginary wavelength, which appears in the field expression as $\exp[-i(\omega t - z/\lambda_g)]$ leading to the exponential decay of the power $\exp(-2z/\lambda_g)$ known as “evanescent waves”.

The earth waveguide between the earth surface and ionosphere at the height of $d_{\text{ion}} = 60$ km during daytime and $d_{\text{ion}} = 100$ km at night gives the cut-off frequency of $TE_1$ mode for the atmospheric waveguide, i.e., $f_{\text{c}} = c/2d_{\text{ion}} = 2.5 - 1.5$ kHz. The ionosphere $100$ mm $(100$ km) above the crust gives $f_{\text{c}} = 1.5$ GHz $(1.5$ kHz). The corresponding frequency of SEMS is indicated within the bracket. The speed of EM waves in a material with $\varepsilon^*$ is roughly $c/\varepsilon^{1/2}$. The cut-off frequency of the crust, $f_{\text{c}}$, for the model crust of salt solution (that of the earth crust is indicated within the bracket) is $f_{\text{c}} = 560$ MHz $(560$ Hz) using $f_{\text{c}} = c/2\varepsilon^{1/2}d$, $\varepsilon^* = 80$ and $d = 50$ mm.

3. Experimental

3.1 Geographical model

A schematic drawing in Fig. 1 shows the $1:10^6$ scaled geographical model of Tokai area, Japan. The upper crust was simulated by water or salt water with an area of $1,200$ mm $\times 1,200$ km $\times 1,200$ km and a depth of $30$ mm $(30$ km). The ionosphere at a height of $100$ mm $(100$ km) and the top surface of the conductive lower crust were simulated by aluminum plates with $\rho = 3 \times 10^{-8}$ $\Omega$ m for reflecting EM waves as earth waveguides and not to simulate them. The conductive ocean was represented by aluminum foils supported by soft sponge blocks soaked in water. The faults were simply simulated by aluminum foils by considering them as active faults with a depth of $20$ mm $(20$ km) and structural lines of $30$ mm $(30$ km) going to the lower crust. The corresponding size in the geographical scale is given in brackets (Fig. 1).

3.2 Experiment of propagation of seismic EM waves

The EM field was assumed to be generated by electric polarization along active faults at the vicinity of the epicenter. Experimentally, EM waves were emitted from a small antenna placed at the model hypocenter connected to a microwave synthesizer (Micro Device, MWSG-18SX) of $400$ MHz $(400$ Hz), $500$ MHz $(500$ Hz) and $2.5$ GHz $(2.5$ kHz). The power intensity of the source was fixed at $15$ dBm $(31.6$ mW).

The detector antenna was another small antenna close to the surface of the NaCl solution with $\rho = 2.0 \times 10^{-2}$ $\Omega$ m which is scanned over the surface by an X-Y plotter (Yokogawa, MP3300) and connected to a spectrum analyzer (Advantest, R3271). The detected area is $200$ mm $\times 200$ mm $(200$ km $\times 200$ km). The microwave synthesizer, X-Y plotter and spectrum analyzer are all controlled by a personal computer and the data are processed to give a map of the intensity profile of EM waves by scanning the detector over the geographical model.

3.3 Experiment of the spatial distribution of electric potential

A classical method of electrostatic potential measurement using the electrolytic tank was used by applying it to a vacuum tube. The land was replaced by water with a depth of $30$ mm $(30$ km). Aluminum foils were used to represent the conductive ocean and the conductive faults. The active faults and the structural lines were set in the model land by considering depths of $20$ mm $(20$ km) and $30$ mm $(30$ km). The AC potential was induced by small electrodes connected to oscillators on both ends of an active fault in the model. A voltage was applied to the electrode using alternating voltage at $5$ Hz. DC potential was avoided to prevent polarization and electrolysis. The detector is a needle probe, placed on the model earth surface and in light contact with it. The scanned surface by an X-Y plotter and the output was connected to the storage oscilloscope (LeCroy 9314AM). The scanned area was $200$ mm $\times 200$ mm $(200$ km $\times 200$ km). The X-Y plotter and storage oscilloscope were controlled by a personal computer and the data were processed to obtain a map of the distribution of the electric potential.
4. Results and Discussion

4.1 Propagation of seismic EM waves

4.1.1 Effect of the distribution of ocean, land and faults

Distribution of the power intensity of the EM waves at 500 MHz in Tokai area is shown in Fig. 2. It was obtained by applying the EM waves with a power of 15 dBm to the source antenna as a hypocenter. The antenna was fixed at 2 mm (2 km) below the surface at the center of the map. Unfortunately, the emission power of 15 dBm was too low to study the distribution due to the detection limit of -99 dBm. Hence, we used an aqueous NaCl solution (10^{-1} \, \text{Ωm}) to model bedrock granite of 10^5 \, \text{Ωm} rather than a KCl solution of 10^{-2} \, \text{Ωm} (bedrock of 10^3 \, \text{Ωm}). The image has a spread of about three times the case of the bedrock of 10^6 \, \text{Ωm}. The SEMS frequency of 500 Hz for simulating 500 MHz is in the extremely low frequency (ELF) range. However, the overall effects of the ocean (geographic selectivity) and the conductive fault planes on the evanescent waves can be estimated from the results simulating 500 Hz. The influence of the presence of ocean is apparent that the strong intensity area of EM waves is almost limited to the land [see Fig. 2(b)]. This is due to the change of radiation pattern of EM waves by ocean and faults that worked like an antenna in the vicinity of the epicenter.

A similar experiment was conducted after removing the aluminum foils representing the ocean part to investigate the ocean influence on the distribution of the electric potential. The intensity of EM waves decayed exponentially [see Fig. 2(a)], which is completely different from the result shown in Fig. 2(c).

4.1.2 Effect of the depth of the source on the model epicenter

Figure 3 shows the attenuation of the power intensity of EM waves at frequencies of 400 MHz (400 Hz) and 2.5 GHz (2.5 kHz) against the depth of the source antenna from the surface of the model crust. This experiment was conducted with and without the model crust composed of KCl solution with \( \rho = 4.8 \times 10^{-2} \, \Omega m \) (4.8 \times 10^5 \, \text{Ωm}) while the depth of the source antenna was varied by 1 mm (1 km) from the surface of the model crust. The solid line and the dotted line indicate the calculated values considering the attenuation of EM waves due to the skin depth and by the distance between the emitter and detector antennae, the circles and stars indicate the experimental values of the attenuation at 400 MHz (400 Hz) and 2.5 GHz (2.5 kHz). It is clear that the experimental attenuation value of EM waves is almost equal to the calculated one. Scaling up the experimental value to the real case, the power intensities of the EM waves decayed by -3.1 dB/km and -5.4 dB/km for 400 Hz and 2.5 kHz respectively. We calculated the attenuation value considering two different dielectric coefficients \( \varepsilon^* = 8 \), and 80 corresponding to those of the earth crust and water. However, there is no clear difference at these frequencies although the dielectric coefficient of the rock was reported to depend on the frequency and the water content.

Fig. 2. Intensity maps of EM waves at 500 MHz (500 Hz for SEMS) below the cut-off frequency, \( f_c \), of earth waveguides. (a) Evanescent waves, (b) the effect of the Pacific Ocean and (c) the conductive faults Tokai area, Japan. Black lines and gray lines indicate active faults and structural lines.

Fig. 3. Attenuation of the power intensity of EM waves at 400 MHz (400 Hz) and 2.5 GHz (2.5 kHz) against the depth of the source antenna from the surface. The solid line and the dotted line indicate the calculated values considering the attenuation of EM waves due to the skin depth and by the distance between the emitter and detector antennae, circles and stars indicate the experimental values of attenuation at 400 MHz (400 Hz) and 2.5 GHz (2.5 kHz).
Japan is shown in Fig. 4. It was obtained by applying a voltage of 5 V at 5 Hz to the electrodes fixed at the center of the map; DC potential was avoided to prevent polarization and electrolysis. It is clear that a high electric field appeared along the coastline and conductive faults disturbed the distribution of the electric potential.

A similar experiment was conducted after removing the aluminum foil representing the ocean part to investigate the ocean influence on the distribution of the electric potential. The electric potential basically decayed from the center of the map [see Fig. 4(a)], which is completely different from the result shown in Fig. 4(c). The result of the experiment indicates that the earth potential is affected by the conductive ocean and therefore the potential is different. The presence of the conductive fault plane also affected the potential, presumably leading to the selectivity in the VAN method.

### 4.3 Scaling experiment to nanosized devices

The present study was a model experiment of the earth scaled down to a laboratory model using a map of Tokai area and the conductive earth crust for studying on earth sciences based on applied physics. It is of interest and also intriguing to consider the same scaling experiment for mesoscopic and nanosized electron or optical waveguides. The conductivity of the material should always be changed so that the scaling law of the Maxwell equation can be satisfied. One might be able to fabricate with great ease a model circuit larger than a nanosized optical circuit using a frequency lower than the real value. The conductivity of the model materials should also be scaled down using more resistive materials so that the skin depth is also scaled up. Although arguments have been presented that a computer model calculation such as a finite element method is feasible for estimating the propagation of EM waves in a complex system including the earth crust, a simple model experiment would give a better insight and new ideas as in the case of cars fabrication and building designs. The same idea can be used for scaling up of micro-, meso- and nanosized devices as long as nonlinear or quantum effects are not involved in the propagation of EM waves. This was demonstrated for a scanning electron spin resonance (ESR) microscope as an analogy to a near-field optical microscope. Microwaves coming from a small aperture with a diameter less than the wavelength in a microwave cavity allow the imaging of the distribution of unpaired spins.10)

### 5. Summary

Model experiments were carried out to demonstrate that an accurate scaling method could be obtained by selecting materials with resistivities in concordance with the scaling factor. The geographic effects of the ocean, land and conductive faults on the propagation of EM waves and the distribution of electric potential were studied taking the geography of the Tokai area into account. The EM waves at a frequency lower than the cut-off frequency, i.e., ELF range, decayed exponentially from the source and were pushed by the presence of the ocean to the side of the land. The conductive faults worked like an antenna plane if the conductive plane was placed at a distance shorter than the wavelength of EM waves from the source. However, the influence of the faults disappeared as the distance increased. The electric potential decayed from the model epicenter and was distorted by the presence of the ocean and the faults. A high electric field appeared along the coastline. The present method of scaling the skin depth could be used for scaling-up experiments of meso- and nanosized optical devices.

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